Experimental Results

The static (d.c.) pressure in the cavity was measured at all three boundary-layer conditions. It was observed that maximum static pressure developed at a given amplitude with no flow ($U_\infty = 0$) was negligible compared to the pressure developed in the presence of a mean flow. If the loudspeaker was not energized ($\hat{A} = 0$) the pressure rise in the cavity was quite small and reproducible. With both flow and oscillations in the cavity it was found that the pressure rise in the cavity increased monotonically with amplitude $\hat{A}$ and the maximum value reached was about 4% of the dynamic pressure.

Clearly, the set of experiments was of a limited range. Nevertheless, one may speculate about possible scaling laws. The displacement amplitude $\hat{A}$ characterizes the magnitude of the oscillation. The appropriate scaling length must depend on both the boundary-layer thickness δ_* and on the diameter of the well D. The simplest assumption would give both these length scales an equal weight with the choice $\sqrt{\delta_* D}$. The previous normalization for a single frequency f resulted in the collapse of all data (Fig.2).

For varying frequencies, it is reasonable to base the normalization on the Strouhal number, defined as $S = fD/U_\infty$. A tentative rough scaling law may be ventured.

$$\Delta p/q = K(\hat{A}/\sqrt{\delta_* D})^{1.4}(fD/U_\infty)^{1.75}$$

where $q = 1/2\ \rho\ U_\infty^2$ and the nondimensional constant $K = 5$. Figure 3 shows that at least the limited data appear to be in fair agreement with the proposed formula.

Discussion

For the explanation of the observed phenomenon the following model is suggested. Due to the oscillation of the air column in the cavity, the boundary layer on the plate is periodically sucked into the cavity and blown out of it. In the suck-in phase a stagnation point is created on the downstream lip of the cavity, and a temporary pressure rise builds up inside the cavity. In the blow-out phase no corresponding negative pressure builds up, so a "rectifier" effect causes the static pressure in the cavity to increase monotonically with the amplitude of the oscillations. At very high amplitudes the pressure increase must reach saturation limit. In many technologically interesting configurations, the static pressure in a cavity may not be assumed equal to the freestream pressure when oscillations are present in the cavity.

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Shock Expansion Analysis of Hypersonic Pistons Decelerating in Long Tubes

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THE existence of light gas guns has made it possible to accelerate light models (0.1 g) to 10,000 km/sec and more massive models to correspondingly lower velocities.[1] Studies of the flow around these high-speed bodies can be made with spark shadowgraph systems, and internal structural damage can be observed with flash x-ray cameras. If the condition of the surface of a model is of interest, the model must be photographed or brought to rest nondestructively. The photography of models requires a high-speed pulsed laser and is difficult. Mechanically stopping the model has been achieved with bales of rags or similar material; however, stopping a model with solid material can drastically alter the model face and obscure experimental details.

The current trend is to attempt to stop a high-speed model by catching it in a tube filled with moderate-pressure gas. The procedure in a typical experiment is to launch the high-speed model with a conventional light gas gun, pass the model through an environment which may disturb its surface (i.e., dust, water drops, etc.), and then to admit the model into a catch tube via a diaphragm section or a fast opening valve.[2] The purpose of the valve is to separate the normally low-pressure test section from a catch tube containing a higher-pressure gas.A simple, long tube at the test section pressure could be used, but would be longer than necessary since most models can stand a higher deceleration than would be generated by maintaining the catch tube at a low pressure.

A shock-expansion analysis is presented here to compute the deceleration of a hypersonic piston in a long tube. If we treat the gas processed by the generated shock as ideal, closed-form solutions are obtained for velocity and position of the piston as a function of time as well as the pressure and temperature at the piston face. The shock wave which runs ahead of the model may be treated as real, but the weakening of the shock due to the model deceleration is ignored. The analytical model is similar to a conventional shock-tube analysis; this treatment differs from the conventional shock-tube analysis in that the model deceleration is caused by the shock pressure, which decays as the model decelerates. Thus, the model dynamics and the gas dynamics are intimately coupled. The following will present an analysis of the decelerating model and the conditions which will occur at its free surface.

Analysis

The affect of the diaphragm rupture or the opening time of a fast valve in the deceleration tube will be presumed to be small. This is justified since stopping distances will usually be large compared to the region disturbed by the opening process.Figure 1 shows the model (or piston) after it has entered the deceleration tube. The piston generates a shock[3] which moves away from it. If the piston velocity were constant, then Fig. 1 would just represent the classical shock-tube problem. In this case the piston is constantly decelerating. As the piston decelerates, expansion waves are generated which weaken the shock, as depicted in Fig. 2. In turn the expansion waves will interact with the shock wave and will generate waves which will return to the piston. Thus, the region between the shock and the piston is filled with a nonsimple wave pattern and must in general be analyzed using the theory of characteristics. Mahoney[4] has shown, for a prescribed piston motion, that shock-expansion theory agrees closely with an exact solution of the theory of characteristics. He found that the entropy gradients produced by the decelerating shock

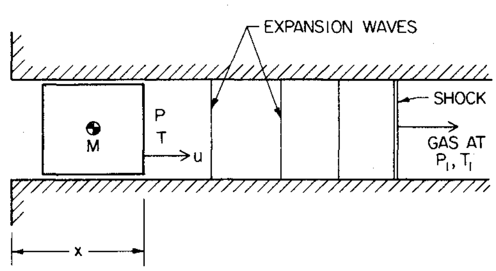

Fig. 1 The model decelerating in a tube. The shock is continuously weakened by expansion moves generated by the piston deceleration.

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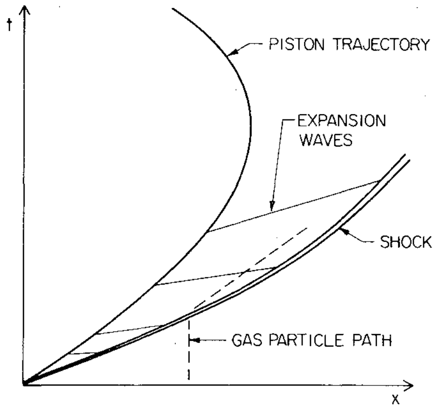

Fig. 2 *x*-t schematic of piston deceleration.

tended to cancel out the effect of waves originating at the shock wave which return to the piston. The ability to ignore the effects of entropy gradients and returning waves allows the region between the piston and the shock to be treated as a simple expansion. Using this allows a unique relationship to exist between the velocity of the piston and the state of the gas just in front of it.

Ignoring friction between the piston and the tube, and the pressure behind the piston, we can write

$$M(du/dt) = -PA \tag{1}$$

where M, A, and u are piston mass, frontal area, and velocity; t is the time after entry into the tube. Shock-expansion theory predicts that the entropy between the piston and the shock is constant and that the Rieman invariant for one-dimensional flow at the piston face is[3]

$$a/a_2 = 1 - [(\gamma-1)/2]\ [(U/a_2) - (u/a_2)] \tag{2}$$

where U is the initial piston speed, a_2 is the sound speed just behind the shock, and a is the sound speed at the piston face. γ is the specific heat ratio between the piston and the shock. Here the gas between the piston and the shock wave is assumed to behave isentropically. The shock wave itself will usually be so strong that real gas effects across the shock must be accounted for.

Letting $u/a_2 = V$ and $\tau = [P_2A/(Ma_2)]t$, Eqs. (1) and (2) plus the ideal isentropic gas law (again this is only assumed to be true for gases between the piston and the shock) yields

$$\mathrm{d}V/\mathrm{d}t = -[1 - N(\gamma-1)/2 + (\gamma-1)/2V/2]^{2\gamma(\gamma-1)} \tag{3}$$

The initial conditions at $\tau = 0$ are $u/a_2 = U/a_2 = N$. Integration yields

$$V = [N - \frac{2}{\gamma-1}] + \frac{2}{\gamma-1}[1 + \frac{\gamma+1}{2}\tau]^{-(\gamma-1)/(\gamma+1)} \tag{4}$$

The trajectory (in the *x-t* plane) of the piston is related to the velocity by

$$V = (1/a_2)(\mathrm{d}x/\mathrm{d}t) \tag{5}$$

where x is the distance from the tube entrance. Again substituting for t and using the definition $Z = x/L$, where $L = Ma_2^2/P_2A$, Eqs. (4) and (5) yield

$$Z = \frac{2}{\gamma-1}[(1 + \frac{\gamma+1}{2}\tau)^{2/\gamma+1} - 1] - (\frac{2}{\gamma-1} - N)\tau \tag{6}$$

using $Z = 0$ when $\tau = 0$. The subscript 2 always refers to conditions just behind the shock at $x = 0$. The minimum deceleration tube length, Z_m, required to stop the piston completely can be obtained by setting $V = 0$ in Eq. (4) and substituting the value of τ in Eq. (6) to get

$$Z_m = \frac{2}{\gamma-1}[(1 - N\frac{\gamma-1}{2})^{-2/(\gamma-1)} - 1]$$

$$-(\frac{2}{\gamma-1} - N)[(1 - N\frac{\gamma-1}{2})^{-(\gamma+1)/(\gamma-1)} - 1]\frac{2}{\gamma+1} \tag{7}$$

The value of N is the same as the induced Mach number in a shock tube where the piston velocity is U. It is well known that for high values of U (or strong shocks in shock tubes) that the maximum value for N for an ideal gas is

$$N_m = -\{2/[\gamma(\gamma-1)]\}^{1/2} \tag{8}$$

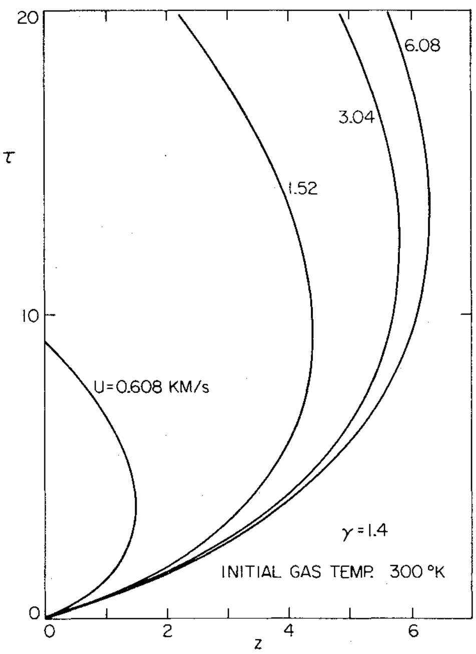

Fig. 3 z-τ trajectory for models decelerating in ideal nitrogen.

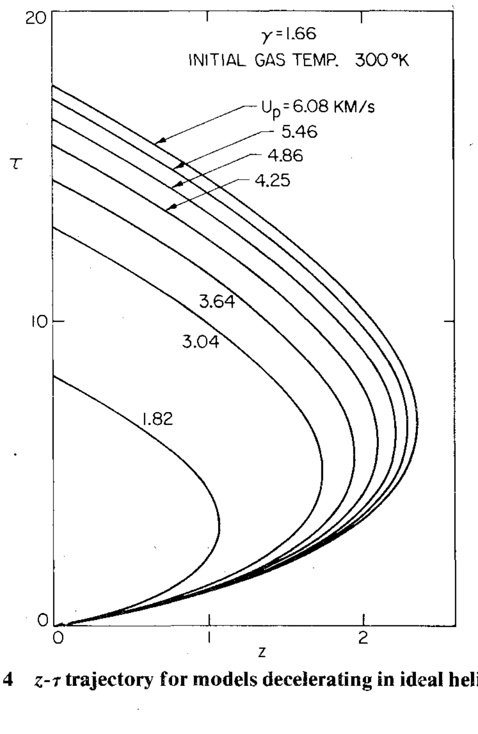

Fig. 4 z-τ trajectory for models decelerating in ideal helium.

Finally, we can obtain the pressure and temperature at the piston face from Eqs. (2) and (4) and the ideal gas equations. The results are

$$T/T_2 = [1 + (\gamma + 1)\tau/2]^{-2(\gamma-1)/(\gamma+1)} \quad (9)$$

and

$$P/P_2 = [1 + (\gamma + 1)\tau/2]^{-2\gamma/(\gamma+1)} \quad (10)$$

where T is the gas temperature.

Figures 3 and 4 show the trajectory of Z vs τ for $\gamma = 1.4$ and $\gamma = 1.66$ plotted against initial piston speed instead of N. As can be seen for large values of U the curves coalesce. This is just due to the fact that N approaches a maximum.

For very-high-speed models real gas effects must be included. This is most simply done by estimating the value of γ behind the shock. For air or easily excitable molecules, γ is as low as 1.1 - 1.15. However, for high-velocity models any oxygen bearing molecule will probably generate an excessive amount of free oxygen, which will react with the model. For deceleration using nitrogen as the stopping gas, γ will be close to 1.35 for realistic model speeds. Using the real shock conditions N can be found as well as P_2, T_2, and a_2 for a given set of initial catch tube pressure and temperature.

Two experimental points are available.[2] In the first, the point of reversal for a 46-g model was measured. Analysis indicates this point should lie near 29.7m, while the actual value is 28.0 m. The velocity for this measurement was 5.03 km/sec, and the gas was nitrogen. In the second piece of available data, the model did not reverse but was caught in a rag bundle after traversal of the tube. The initial velocity was 2.55 km/sec and the theory predicts a total stopping length of about 30 m. The model was traveling slowly enough to be easily captured in the rag bundle after a deceleration length of 33 m with no evident damage.

Care must be exercised in the comparison of data with the analysis since all the variables are sensitive to γ and the sound speed a_2. Values as close to actual as possible must be used to obtain good comparison.

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Correlation for the Viscosity of Air Including Effects of Dissociation

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A METHOD with which the viscosity of air can be simply and accurately calculated between a match point with the Sutherland formula at 1050°R and the enthalpy level, 20,000 Btu/1bm, at which ionization is about to become significant is presented here. The suggested expression, which is a function of temperature and the degree of dissociation, is a correlation of the results of Clark, et al.[1] That analysis of the transport properties of air is described briefly in the following paragraphs.

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In the investigation of Ref. 1, the thermodynamic and transport properties of both air and pure nitrogen were calculated for pressures in the range 1-200 atm and temperatures in the range 1000-30,000K. This was done because available and well-know calculations[2-6] do not extend over the range of pressures and temperatures of interest, or they are based on outdated collision integrals. The present discussion is confined to the calculations for temperatures below 10,000K, which is the regime of interest in the present note. The thermodynamic properties calculated included the species mole fractions which are needed to compute the viscosity and other transport properties. The Aerotherm Chemical Equilibrium (ACE) computer program was used to calculate these species mole fractions, assuming the air system to be comprised of the species N_2, N, N^+, O_2, O, O^+, NO, NO^+, and e^-. The ACE calculations were compared with the tabulated results of Hilsenrath and Klein,[7] and were found to be within a few per cent at all times.

The transport properties were calculated using the mixture rules of Yos.[2] These expressions reduce to the results of rigorous kinetic theory in the limit of a one-specie gas. For mixtures, they are approximate in that they exclude the higher-order terms in the first Chapman-Enskog approximation. However, calculations based on the simpler mixture rules rarely differ from the more exact first approximation by more than a few per cent.[2] A literature review was conducted to determine the best set of collision integrals to use in conjunction with the Yos mixture expressions. The collision integrals presented in the original work of Yos were used whenever they were confirmed through the literature survey. However, a number of these integrals required updating (see Ref. 1 for details).

Using the formulation just described, we calculated the viscosity, and in addition, frozen and reactive thermal conductivity and electrical conductivity for air and pure nitrogen. The results of these calculations were compared critically with numerous experimental measurements of transport properties in the temperature range 6000-24,000K.[8-13] These data were acquired from experiments utilizing cascade electric arcs at 1 atm pressure. In general, the calculated transport properties agreed with the various measurements to within their associated experimental uncertainties. In addition, the present calculations were compared with several heavily referenced calculations available in the literature for the pressure range 1-100 atm.[2-6] In general, the present results and those of Peng and Pindroh[6] are in good agreement for all pressures and temperatures of interest. However, Hansen's[5] results, based on outdated collision integrals, deviate considerably from the present results.[1] In summary, it is felt that the calculations presented in Ref. 1 represent the best-validated and most current state-of-the-art air transport properties now available. They, therefore, provide a sufficiently firm foundation for the viscosity correlation presented herewith.

A somewhat different approximate kinetic theory analysis is employed in the Boundary Layer Integral Matrix Program (BLIMP code).[14-15] That analysis indicates that, for a mixture of fixed composition, the viscosity is well represented by a power law dependence upon temperature.

$$\mu/\mu_0 = (T/T_0)^{\omega}$$

This expression, with $\omega = 0.659$ (which is the value used in the BLIMP code) and with a modification which takes into account the effects of dissociation, is used here to correlate the air viscosities calculated by Clark.